Fiscal-Monetary Interactions and the FTPL:

"Paper Money"
Sims (2013)

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Topics in Macroeconomics A

uc3m, spring 2025

Motivation

- Recent developments in CB balance sheets and sovereign debt sizes
- Fiscal and monetary policy are deeply intertwined
- Conventional (quantity theory) models with
  - non-interest bearing money
  - a "money multiplier"
  - tight relation between P and M

  are inadequate for *current* policy discussions
- The FTPL is a more adequate framework
  - *this paper tries to bring FTPL down to earth*

First model

Samuelson's consumption loan model with storage

Simple OLG model with gov't debt and storage

- Households

$$\max_{\{c_t^y, c_{t+1}^o, B_t, s_t\}} \quad \log(c_t^y) + \log(c_{t+1}^o)$$

$$\text{s.t.} \quad c_t^y + s_t + \frac{B_t}{P_t} = e^y$$

$$c_{t+1}^o = \frac{B_t R}{P_{t+1}} + \theta s_t, \quad \theta \in (0,1)$$

- Government

$$B_{t+1} = RB_t$$

$$B_t \geq 0$$

can always think of $R = 1$ and of debt as paper money

Optimality

$$\frac{c^o_{t+1}}{c^y_t} = \theta \quad \text{if } s_t > 0$$
$$\frac{c^o_{t+1}}{c^y_t} = R_t \frac{P_t}{P_{t+1}} \quad \text{if } B_t > 0$$

Let

- $W_t := s_t + \frac{B_t}{P_t}$ denote savings
- ρ_t be the real rate of return on W_t

The log-utility assumption implies

$$\frac{c^o_{t+1}}{c^y_t} = \rho_t \quad \Rightarrow \quad \frac{\rho_t W_t}{e^y - W_t} = \rho_t \quad \Rightarrow \quad W_t = c^y_t = e^y/2$$

Equilibrium without storage

Households only save in bonds $W_t = \frac{B_t}{P_t} = e^y/2$

From the goods market clearing condition

$$c_t^y + c_t^o = e^y$$
$$e^y/2 + \rho_t e^y/2 = e^y$$

which implies $\rho_t = 1$, $R = \frac{P_{t+1}}{P_t}$, and $c_{t+1}^o = c_t^y = e^y/2$

The government budget implies that the real value of debt is constant

$$\frac{B_t}{P_t} = \frac{B_{t-1}}{P_{t-1}}$$

and the debt market clearing condition requires it is equal to household savings: $\frac{B_t}{P_t} = e^y/2$

Equilibrium without storage

Consumption of the initial old is given by

$$c_1^o = R\frac{B_0}{P_1} = \frac{B_1}{P_1} = e^y/2$$

so that

$$P_1 = \frac{2}{e^y}RB_0.$$

The price level is uniquely determined.

Equilibrium with storage

By no-arbitrage, $\theta = \rho_t = R\frac{P_t}{P_{t+1}}$ (the inflation rate $\frac{P_{t+1}}{P_t} = \frac{R}{\theta}$ is higher now)

Plugging no-arbitrage into the govt BC

$$\frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}}$$

so in the limit $\frac{B_t}{P_t} \to 0$ and in turn $s_t \to e^y/2$

In the initial period, storage and real debt are indeterminate. Any

$$c_1^o = R\frac{B_0}{P_1} = \frac{B_1}{P_1} < e^y/2 \quad \Leftrightarrow \quad P_1 > \frac{2}{e^y} RB_0$$

is an equilibrium. **The price level is indeterminate.**

Note: in any equilibrium with storage, $c_t^o = \theta/2 < 1/2$ for all t, worse than no-storage eqm

Discussion

Remember that we can always think of paper money if $R = 1$ and $B \equiv M$.

In the equilibria with storage, P_1 is "too high"

- there is too little real debt available for households to save
- they then use storage, rates of return are low because of no-arbitrage,
- government pays negative interest rates (runs surpluses!), future real debt is even scarcer, and so on...

Tax Backing. Now, assume that the young pay lump-sum taxes

$$c_t^y + W_t + \tau = e^y$$

$$\frac{B_t}{P_t} = R\frac{B_{t-1}}{P_t} - \tau$$

Equilibrium with storage and tax backing

Recall that by no-arbitrage $\rho_t = \theta < 1$, which implies $\frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}}$

Iterate gBC backwards

$$\frac{B_t}{P_t} = \theta^{t-1}\frac{B_1}{P_1} - \tau \sum_{j=0}^{t-1} \theta^j \quad \text{so that} \quad \lim_{t\to\infty} \frac{B_t}{P_t} = -\frac{\tau}{1-\theta}$$

so $\rho_t < 1$ cannot be an equilibrium: in the limit, gov't would be net saver, which we are ruling out ($B_t \geq 0$). Intuitively,

- the gov't surpluses now are independent of the size of debt, so it eventually accumulates savings $\Rightarrow$ fiscal policy now incompatible with arbitrary path of prices
- If $B \equiv M$, the gov't is shrinking the stock of money by raising taxes

In either case, household wealth eventually not enough to finance taxes. The demand for savings $\uparrow$, $P_1 \downarrow$.

Equilibrium without storage and tax backing

Same idea as equilibrium without tax backing, but now youngs have smaller effective endowment $e^y - \tau$

- lower savings $W_t = \frac{e^y - \tau}{2}$
- higher real rate of return $\rho_t = \frac{e^y + \tau}{e^y - \tau} > 1$
- less consumption smoothing: $c_t^y = \frac{e^y - \tau}{2}$, $c_{t+1}^o = \frac{e^y + \tau}{2}$
- for the initial old, $c_1^o = \frac{RB_0}{P_1} = \tau + \frac{B_1}{P_1} = \tau + \frac{e^y - \tau}{2}$ so that $P_1 = \frac{2}{e_y - \tau} RB_0$

The government budget is $\frac{B_t}{P_t} = \frac{B_{t-1}}{P_{t-1}}\rho - \tau$, and real debt is constant. The debt valuation equation holds: $\frac{B_t}{P_t} = \frac{\tau}{\rho - 1}$

Taking stock

Without fiscal backing

- 1 eqm without storage, where govt paper is valued as a store of value, and $1 = R\frac{P_t}{P_{t+1}}$ (Wallace (1998): use of money as endogenous outcome rather than assumption)
- ∞ eqa with storage

Govt paper can have value in these models even if it is not backed

With fiscal backing

- equilibria with indeterminate P_t *and* $B_t/P_t \to 0$ are ruled out
- unique eqm has lower welfare, but arbitrarily close to perfect smoothing as $\tau \to 0$, and $\frac{e^y+\tau}{e^y-\tau} = R\frac{P_t}{P_{t+1}}$

Note: you have seen case with no storage technology, $B_t = M$ and $e^o > 0$, which also had

- ∞ eqa where money is valued but its value converges to zero ($P_t \to \infty$)
- autarky eqm where money is never valued

Second model

Debt as fiscal cushion

Well-known optimal fiscal-monetary policy results:

1. With distortionary taxes and state-contingent debt, taxes are smooth and independent of the debt stock, and debt returns absorb shocks (Lucas and Stokey (1983))

2. Surprise inflation can make non-contingent nominal debt state-contingent in real terms
   - but that is only optimal when surprise inflation is costless (Siu (2004), Schmitt-Grohé and Uribe (2004))
   - with long-term debt, state-contingency can be achieved through debt valuation effects (i.e. future inflation)

Debt as fiscal cushion

This model

- adds price level determination to Barro (1979)
- shows how nominal debt can be used as a "fiscal cushion" via long-term interest rates and/or inflation

Govt objective

$$\max_{P_t, B_t, R_t, \tau_t} -\frac{1}{2}\mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \left(\tau_t^2 + \theta(\nu_t - 1)^2\right)\right]$$
$$\text{s.t.} \quad b_t = R_{t-1}\nu_t b_{t-1} + g_t - \tau_t$$
$$R_t \mathbb{E}_t[\nu_{t+1}] = \rho$$

with $\nu_t = P_{t-1}/P_t$, $b_t = B_t/P_t$ and $\rho = 1/\beta$
g_t is exogenous and random

Optimality

First-order conditions

$$\tau_t = \lambda_t \quad \text{(taxes)}$$
$$\lambda_t = \beta R_t \mathbb{E}[\nu_{t+1}\lambda_{t+1}] \quad \text{(debt)}$$
$$\mu_t \mathbb{E}[v_{t+1}] = \beta b_t \mathbb{E}[\nu_{t+1}\lambda_{t+1}] \quad (R_t)$$
$$\theta(\nu_t - 1) + \lambda_t R_{t-1} b_{t-1} = \mu_{t-1} R_{t-1} \beta^{-1} \quad (\nu_t)$$

Combine (R_t) and (debt): $\mu_t \rho = b_t \lambda_t$

Combining FOCs for b, R, ν we get tradeoff for ν_t

$$\theta(\nu_t - 1) = (\tau_{t-1} - \tau_t) R_{t-1} b_{t-1}$$

welfare loss at t = budget benefit at $(t-1)$ (lower R_{t-1} via Fisher eq.) $-$ budget cost at t

With $\theta = 0$

- $\tau_t = \tau_{t-1} = \tau$ constant
- iterating the govt BC forward we get

$$b_t = \frac{\tau}{\rho - 1} - \mathbb{E}_t \sum_{j=1}^{\infty} \beta^j g_{t+j}$$

- with g_t i.i.d., b_t remains constant
- surprise inflation (swings in ν_t) absorb *all* effect of g_t shocks

With $\theta = \infty$

- $\nu_t = 1$
- $\tau_t = \mathbb{E}_t[\tau_{t+1}]$ (martingale as in Barro (1979))

With $0 < \theta < \infty$

- mix of surprise inflation and tax changes
- compare 1-period with consol debt model

Consol Debt

- let A_t be a consol: never matures, pays 1 dollar every period, has price Q_t
- new govt BC

$$Q_t \frac{A_t - A_{t-1}}{P_t} = \frac{A_{t-1}}{P_t} + g_t - \tau_t$$

- define $b_t := \frac{Q_t A_t}{P_t}$ (real value of consol debt)

$$b_t = b_{t-1} \nu_t \frac{1 + Q_t}{Q_{t-1}} + g_t - \tau_t$$

- Fisher equation of the private sector

$$\mathbb{E}_t \frac{(1 + Q_{t+1}) \nu_{t+1}}{Q_t} = \rho$$

Optimal response to a spending shock

Numerical example (local approximation around steady-state)
g_t i.i.d. with $\mathbb{E}[g_t] = 1$, $\rho = 1.1$, $\tau = 2$, $\nu = 1$, $b = 10$

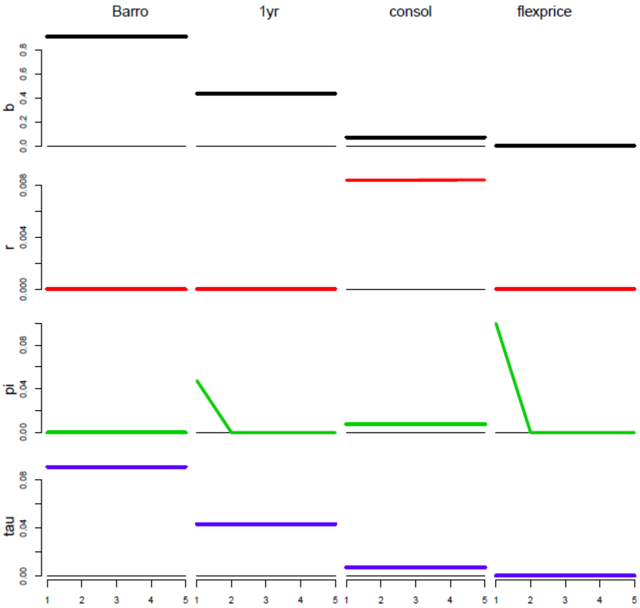
Experiment: one time shock, $\uparrow g_t$ by 1 unit. Study optimal fiscal/monetary policy responses

Real debt ($\theta = \infty$): permanent increase of τ (0.91) and b (0.09)
Increase in τ perfectly smoothed over time, enough to service higher debt forever

Flexible prices ($\theta = 0$): one-off surprise $\uparrow \pi$ by $10p.p.$ ($\approx$ small default)
Small one-off reduction in debt service, nothing else changes

Intermediate case ($\theta = 10$):

- One-Year Debt
  - permanent fiscal adjustment ($b \uparrow 0.43, \tau \uparrow 0.043$), one-off monetary ($\frac{1}{\nu} \uparrow 0.048$ p.p.)
  - mainly fiscal response, π-default must be immediate so cannot be too large
- Consol Debt
  - both adjustments permanent ($b \uparrow 0.07, \tau \uparrow 0.007$ and $\frac{1}{\nu} \uparrow 0.74$ p.p.)
  - mainly monetary response, π-default on bondholders spread out to infinity



Barro
1yr
consol
flexprice
b
r
pi
tau
0.0
0.2
0.4
0.6
0.8
0.000
0.004
0.008
0.00
0.04
0.08
0.00
0.04
0.08
1
2
3
4
5

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